SURVIVAL PROJECTION MODEL
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N_{HM_A} = number of hatchlings given adult mortility is equal to that of juveniles and sub-adults.
N_A = number of adults
N_{A1} = number of turtles entering adult group
M_A = instantaneous mortality rate of adults
t_{A1} = age at which turtles reach adulthood
t_{max} = maximum age of turtles
N_{min} = number of turtles left at maximum age
N_{SA1} = number of turtles entering sub-adult group
t_{SA1} = age at which turtles enter sub-adult group
M_{SA} = instantaneous mortality rate of sub-adults
N_{SA} = number of sub-adults
N_H = number of hatchlings entering water
N_J = number of juveniles
M_J = instantaneous mortality rate of juveniles

Using the above notation and the following formulae, N_J, N_{SA} and the mortality rates (M_J, M_{SA}, and M_A) can be calculated.

$$N_A = N_{HM_A} \int_{t_{A1}}^{t_{max}} e^{-M_A t} \, dt$$

$$N_A = N_{A1} \, e^{M_A t_{A1}} \int_{t_{A1}}^{t_{max}} e^{-M_A t} \, dt$$

$$N_A = \frac{N_{A1} \, e^{M_A t_{A1}}}{-M_A} \left[e^{-M_A t_{max}} - e^{-M_A t_{A1}} \right]$$

$$N_A = \frac{N_{A1}}{M_A} \left[1 - e^{-M_A (t_{max} - t_{A1})} \right]$$

$$\frac{1}{N_{A1}} = \frac{1 - e^{-M_A (t_{max} - t_{A1})}}{M_A \, N_A}$$

$$N_{A1} = \frac{M_A \, N_A}{1 - e^{-M_A (t_{max} - t_{A1})}} \qquad (1)$$

If $N_A = N_{min}$ at t_{max} then

$$N_{A1} e^{-M_A (t_{max} - t_{A1})} = N_{min}$$

$$N_{A1} = N_{min} e^{M_A (t_{max} - t_{A1})} \qquad (2)$$

From (1) and (2) we have

$$\frac{M_A N_A}{1 - e^{-M_A (t_{max} - t_{A1})}} = N_{min} e^{M_A (t_{max} - t_{A1})}$$

$$M_A N_A = N_{min} \left[e^{M_A (t_{max} - t_{A1})} - 1 \right]$$

$$M_A N_A - N_{min} e^{M_A (t_{max} - t_{A1})} + N_{min} = 0 \qquad (3)$$

N_A, t_{A1}, and t_{max} are known (or assumed) and M_A can be found numerically using Newton's Method from (3) - then N_{A1} can be obtained from (2).

Assuming subadult mortility (M_{SA}) is a function of adult mortality (M_A) such as

$M_{SA} = \gamma M_A$ where γ is a constant, then

$$N_{SA1} = N_{A1} e^{M_{SA} (t_{A1} - t_{SA1})}$$

then

$$N_{SA} = N_{SA1} e^{M_{SA} t_{SA1}} \int_{t_{SA1}}^{t_{A1}} e^{- M_{SA} t} dt$$

$$N_{SA} = \frac{N_{SA1} e^{M_{SA} t_{SA1}}}{- M_{SA}} \left[e^{- M_{SA} t_{A1}} - e^{-M_{SA} t_{SA1}} \right]$$

$$N_{SA} = \frac{N_{SA1}}{M_{SA}} \left[- e^{- M_{SA} (t_{A1} - t_{SA1})} + 1 \right]$$

Then mortality for juveniles (M_J) can be found from

$$N_H \quad e^{-M_J \; t_{SA1}} = N_{SA1}$$

$$\ln(N_H) - M_J \; t_{SA1} = \ln \; N_{SA1}$$

$$M_J = \frac{\ln(N_H) - \ln N_{SA1}}{t_{SA1}}$$

Then the number of juveniles (N_J) is

$$N_J = N_H \int_0^{t_{SA1}} e^{-M_J \, t} \, dt$$

$$N_J = \frac{N_H}{-M_J} \left[e^{-M_J \; t_{SA1}} - 1 \right]$$

$$N_J = \frac{N_H}{M_J} \left(1 - e^{-M_J \; t_{SA1}} \right)$$

Newton's Method

$F(M_A)$ is a specified function of the adult mortality rate, i.e.,

$$F(M_A) = M_A N_A - N_{min} e^{M_A (t_{max} - t_{A1})} + N_{min}$$

$$F'(M_A) = N_A - N_{min} (t_{max} - t_{A1}) e^{M_A (t_{max} - t_{A1})}$$

$$M_{A_{n=1}} = M_{A_n} - \frac{F(M_A)}{F'(M_A)} \qquad (4)$$

where n is the interation. Recursion equation (4) is repeated until the solution (M_A) converges.